1 a The matrix of the transformation is

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

Therefore, if (x', y') be the coordinates of the image of (x, y), then,

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ -y \end{bmatrix}.$$

Therefore, $x' = x$ and $y' = -y$.
Rearranging gives $x = x'$ and $y = -y''$. Therefore, $y = 3x + 1$ becomes, $-y'' = 3x' + 1$ $y' = -3x' - 1.$ We now ignore the apostrophes, so that the transformed equation is

$$y = -3x - 1.$$

b The matrix of the transformation is

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}.$$

Therefore, if (x', y') be the coordinates of the image of (x, y), then,

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x \\ y \end{bmatrix}.$$

Therefore, $x' = 2x$ and $y' = y$.
Rearranging gives $x = \frac{x'}{2}$ and $y = y''$. Therefore $y = 3x + 1$ becomes,

$$y' = \frac{x'}{2} + 1.$$

We now ignore the apostrophes, so that the transformed equation is

$$y = \frac{x}{2} + 1.$$

c The matrix of the transformation is

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}.$$

Therefore, if (x', y') be the coordinates of the image of (x, y), then,

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x \\ 3y \end{bmatrix}.$$

Therefore, $x' = 2x$ and $y' = 3y$.
Rearranging gives $x = \frac{x'}{2}$ and $y = \frac{y'}{3}$. Therefore $y = 3x + 1$ becomes, $\frac{y'}{3} = 3\left(\frac{x'}{2}\right) + 1$ $y' = \frac{9x'}{2} + 3.$ We now ignore the apostrophes, so that the transformed equation is

$$y = \frac{9x}{2} + 3.$$

d The matrix of the transformation is

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$

Therefore, if (x', y') be the coordinates of the image of (x, y), then,

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ -y \end{bmatrix}.$$

Therefore, $x' = -x$ and $y' = -y$.

Rearranging gives $x = -x'$ and $y = -y'$. Therefore $y = 3x + 1$ becomes,

$$\begin{aligned} -y' &= 3(-x') + 1 \\ -y' &= -3x' + 1 \\ y' &= 3x' - 1. \end{aligned}$$

We now ignore the apostrophes, so that the transformed equation is

$$y = 3x - 1.$$

e The matrix of the transformation is

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$

Therefore, if (x', y') be the coordinates of the image of (x, y), then,

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ 3y \end{bmatrix}.$$

Therefore, $x' = -x$ and $y' = 3y$.

Rearranging gives $x = -x'$ and $y = \frac{y'}{3}$. Therefore $y = 3x + 1$ becomes,

$$\begin{aligned} \frac{y'}{3} &= 3(-x') + 1 \\ \frac{y'}{3} &= -3x' + 1 \\ y' &= -9x' + 3. \end{aligned}$$

We now ignore the apostrophes, so that the transformed equation is

$$y = -9x + 3.$$

f The matrix of the transformation is

$$\begin{bmatrix} \cos 90^\circ & -\sin 90^\circ \\ \sin 90^\circ & \cos 90^\circ \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

Therefore, if (x', y') be the coordinates of the image of (x, y), then,

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ x \end{bmatrix}.$$

Therefore, $x' = -y$ and $y' = x$.

Rearranging gives $x = y'$ and $y = -x'$. Therefore $y = 3x + 1$ becomes,

$$\begin{aligned} -x' &= 3y' + 1 \\ 3y' &= -x' - 1 \\ y' &= \frac{-x' - 1}{3}. \end{aligned}$$

We now ignore the apostrophes, so that the transformed equation is

$$y = \frac{-x - 1}{3}.$$

g Firstly, the rotation matrix is

$$\begin{bmatrix} \cos(-90^\circ) & -\sin(-90^\circ) \\ \sin(-90^\circ) & \cos(-90^\circ) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

The reflection matrix is

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

Therefore, the matrix of the composition transformation is

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

Therefore, if (x', y') be the coordinates of the image of (x, y), then,

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ x \end{bmatrix}.$$

Therefore, $x' = y$ and $y' = x$. Therefore, $y = 3x + 1$ becomes,

$$\begin{aligned} x' &= 3y' + 1 \\ 3y' &= x' - 1 \\ y' &= \frac{x' - 1}{3}. \end{aligned}$$

We now ignore the apostrophes, so that the transformed equation is

$$y = \frac{x - 1}{3}.$$

2 a If (x', y') be the coordinates of the image of (x, y), then $x' = 2x$ and $y' = 3y$. Rearranging gives $x = \frac{x'}{2}$ and $y = \frac{y'}{3}$. Therefore $y = 2 - 3x$ becomes,

$$\begin{aligned} \frac{y'}{3} &= 2 - 3\left(\frac{x'}{2}\right) \\ y' &= 6 - \frac{9x'}{2}. \end{aligned}$$

We now ignore the apostrophes, so that the transformed equation is

$$y = 6 - \frac{9x}{2}.$$

b If (x', y') be the coordinates of the image of (x, y), then $x' = -y$ and $y' = x$. Rearranging gives $x = y'$ and $y = -x'$. Therefore, $y = 2 - 3x$ becomes,

$$\begin{aligned} -x' &= 2 - 3y' \\ 3y' &= x' + 2 \\ y' &= \frac{x' + 2}{3}. \end{aligned}$$

We now ignore the apostrophes, so that the transformed equation is

$$y = \frac{x + 2}{3}.$$

c Let (x', y') be the coordinates of the image of (x, y). Then this transformation can be written in matrix form as:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

Therefore, $$\begin{aligned} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} x' \\ y' \end{bmatrix} \\ &= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix} \\ &= \begin{bmatrix} x' + 2y' \\ y' \end{bmatrix} \end{aligned}$$

so that $x = x' + 2y'$ and $y = y'$. Therefore, $y = 2 - 3x$ becomes $y' = 2 - 3(x' + 2y')$. We solve the equation for y' in terms of x',

$$\begin{aligned} y' &= 2 - 3(x' + 2y') \\ y' &= 2 - 3x' - 6y' \\ 7y' &= 2 - 3x' \\ y' &= \frac{2 - 3x'}{7}. \end{aligned}$$

The transformed equation is

$$y = \frac{2 - 3x}{7}.$$

d Let (x', y') be the coordinates of the image of (x, y). Then this transformation can be written in matrix form as:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

Therefore,

$$\begin{aligned} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} x' \\ y' \end{bmatrix} \\ &= \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix} \\ &= \begin{bmatrix} 2x' - 5y' \\ -x' + 3y' \end{bmatrix} \end{aligned}$$

so that $x = 2x' - 5y'$ and $y = -x' + 3y'$.

Therefore, $y = 2 - 3x$ becomes $-x' + 3y' = 2 - 3(2x' - 5y')$. We solve the equation for for y' in terms of x',

$$\begin{aligned} -x' + 3y' &= 2 - 3(2x' - 5y') \\ -x' + 3y' &= 2 - 6x' + 15y' \\ 12y' &= 5x' - 2 \\ y' &= \frac{5x' - 2}{12}. \end{aligned}$$

fixed

3 There are many answers. We find a matrix that maps the x-intercept of the first line to the x-intercept of the second line, and likewise for the y-intercepts. Then

$$(1, 0) \to (2, 0) \text{ and } (0, 1) \to (0, 2).$$

Since we have found the images of the standard unit vectors, the matrix that will achieve this result is

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}.$$

4 There are many answers. Let's find the matrix that maps the x-intercept of the first line to the x-intercept of the second line, and likewise for the y-intercepts. Then

$$(-1, 0) \to (3, 0) \text{ and } (0, 1) \to (0, 6).$$

The matrix that will achieve this results is

$$\begin{bmatrix} -3 & 0 \\ 0 & 6 \end{bmatrix}.$$

5 Let (x', y') be the coordinates of the image of (x, y). Then the rule for the transformation is given by

$$\begin{aligned} \begin{bmatrix} x' \\ y' \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \left(\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix} \right) \\ &= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x - 1 \\ y + 2 \end{bmatrix} \\ &= \begin{bmatrix} x - 1 \\ -y - 2 \end{bmatrix} \end{aligned}$$

Therefore,
$x' = x - 1$ and $y' = -y - 2$ so that $x = x' + 1$ and $y = -y' - 2$.

Therefore, the equation $y = x^2 - 1$ becomes $-y' - 2 = (x' + 1)^2 - 1$.

Therefore, t $-y' - 2 = (x' + 1)^2 - 1$

$$\begin{aligned} -y' &= (x' + 1)^2 + 1 \\ y' &= -(x' + 1)^2 - 1 \end{aligned}$$

The transformed equation is

$$y = -(x + 1)^2 - 1.$$

6 Let (x', y') be the coordinates of the image of (x, y). Then the rule for the transformation is given by

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$ Therefore,

$$= \begin{bmatrix} -x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$
$$= \begin{bmatrix} -x+2 \\ y-3 \end{bmatrix}$$
$$x' = -x + 2 \text{ and} y' = y - 3$$

so that

$$x = -x' + 2 \text{ and } y = y' + 3.$$

Therefore, the equation $y = (x-1)^2$ becomes $y' + 3 = (-x' + 2 - 1)^2$. Therefore,

$$\begin{aligned} y' + 3 &= (-x' + 2 - 1)^2 \\ y' &= (-x' + 1)^2 - 3 \\ &= (-(x' - 1))^2 - 3 \\ &= (x' - 1)^2 - 3 \end{aligned}$$

The transformed equation is

$$y = (x-1)^2 - 3.$$

7 The dilation matrix is given by

$$\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}.$$

The rotation matrix is given by

$$\begin{bmatrix} \cos 90^\circ & -\sin 90^\circ \\ \sin 90^\circ & \cos 90^\circ \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

Let (x', y') be the coordinates of the image of (x, y). Then the rule for the transformation is given by

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 & -3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3y \\ x \end{bmatrix}.$$

Therefore,

$$x' = -3y \text{ and } y' = x,$$

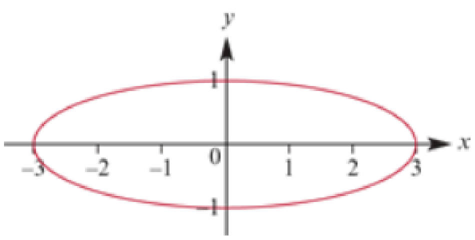
so that

$$y = -\frac{x'}{3} \text{ and } x = y'.$$

Therefore, the equation $x^2 + y^2 = 1$ becomes $(y')^2 + \left(-\frac{x'}{3}\right)^2 = 1$. Ignoring the apostrophes gives,

$$\frac{x^2}{3^2} + y^2 = 1,$$

which is the equation of an ellipse, shown below.



8 Let (x', y') be the coordinates of the image of (x, y). Then this transformation can be written in matrix form as:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

Therefore, $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} \begin{bmatrix} x' \\ y' \end{bmatrix}$

$$= \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{dx'-by'}{ad-bc} \\ \dfrac{-cx'+ay'}{ad-bc} \end{bmatrix}$$

so that

$$x = \frac{dx'-by'}{ad-bc} \text{ and } y = \frac{-cx'+ay'}{ad-bc}.$$

Therefore $px+qy=r$ becomes,

$$p\frac{dx'-by'}{ad-bc} + q\frac{-cx'+ay'}{ad-bc} = r.$$

which, although horribly ugly, is most definitely the equation of a line.

9 The matrix of the transformation is

$$\begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix} = \begin{bmatrix} \dfrac{1}{\sqrt{2}} & -\dfrac{1}{\sqrt{2}} \\ \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \end{bmatrix}.$$

Let (x', y') be the coordinates of the image of (x, y). Then this transformation can be written in matrix form as:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \dfrac{1}{\sqrt{2}} & -\dfrac{1}{\sqrt{2}} \\ \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}.$$

Therefore,

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \dfrac{1}{\sqrt{2}} & -\dfrac{1}{\sqrt{2}} \\ \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \end{bmatrix}^{-1}\begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \\ -\dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix} x'+y' \\ -x'+y' \end{bmatrix}$$

so that

$$x = \frac{1}{\sqrt{2}}(x'+y'),$$

$$y = \frac{1}{\sqrt{2}}(-x'+y').$$

Therefore, $y = \dfrac{1}{x}$ becomes,

$$\frac{1}{\sqrt{2}}(-x'+y') = \frac{1}{\frac{1}{\sqrt{2}}(x'+y')}.$$

Ignoring the apostrophes, and simplifying this expression gives,

$$\frac{1}{\sqrt{2}}(x+y)\frac{1}{\sqrt{2}}(-x+y) = 1$$

$$\frac{1}{2}(x+y)(y-x)=1$$
$$y^2-x^2=2$$
This is the required equation.